The exceptional group G_4

1 Presentation

The group G_4 admits the following presentation:

$$G_4 = \langle s_1, s_2 \mid s_1^3 = s_2^3 = 1,\ s_1 s_2 s_1 = s_2 s_1 s_2 \rangle.$$

The Hecke algebra associated with G_4 admits the following presentation:

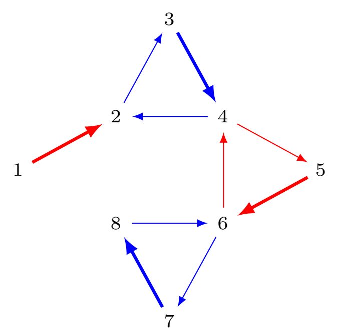
$$H(G_4) = \langle \sigma_1, \sigma_2 \mid \sigma_1\sigma_2\sigma_1 = \sigma_2\sigma_1\sigma_2, \quad \sigma_i^3 = a\sigma_i^2 + b\sigma_i + c, \quad i = 1, 2 \rangle.$$

2 The parabolic subgroup

Let W' the parabolic subgroup generated by s_1.

3 (Partial) coset graph

Let $w = \sigma_1\sigma_2\sigma_1$.
The anchor cosets are: $x_{2k+1} = w^k$, $k = 0, \ldots, 3$.
And the spanning tree: $x_2 = \sigma_2$.
The (partial) coset graph is the following:



4 The choice of w_C

Let $\{b_i\}$ be the basis of the Hecke algebra (we explain how we find this basis in the corresponding article and one can see it explicitly written in the GAP program, which corresponds to G_4).

The indices i of the b_i representing the conjugacy classes of G_4 are

$$1, 13, 24, 15, 23, 10, 22$$

in the order of the conjugacy classes in CHEVIE.

5 The canonical trace

Define $\tau\colon H(G_4) \to R$ as $\tau(\sum \alpha_i b_i) = \alpha_1$. Further calculations yield: $\det(\tau(b_i b_j)) = -c^{96}$.