The exceptional group G_{12}

1 Presentation

The group G_{12} admits the following presentation:

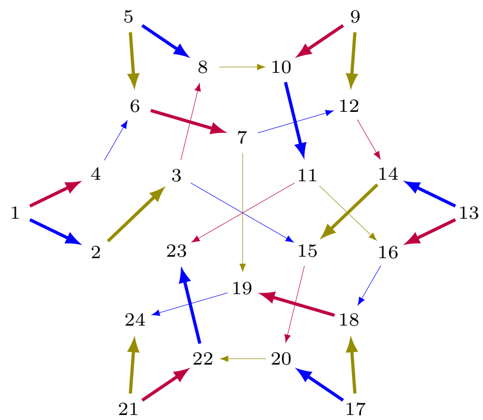
$$G_{12} = \langle s_1, s_2, s_3 \mid s_1^2 = s_2^2 = s_3^2 = 1,\ s_1s_2s_3s_1 = s_2s_3s_1s_2 = s_3s_1s_2s_3\rangle.$$

The Hecke algebra associated with G_{12} admits the following presentation:

$$H(G_{12}) = \langle \sigma_1, \sigma_2, \sigma_3 \mid \sigma_1\sigma_2\sigma_3\sigma_1 = \sigma_2\sigma_3\sigma_1\sigma_2 = \sigma_3\sigma_1\sigma_2\sigma_3, \quad \sigma_i^2 = a\sigma_i + b, \quad i = 1, 2, 3\rangle.$$

2 (Partial) action coset graph

Let $w = \sigma_1\sigma_2\sigma_3\sigma_1$.
Let W' the parabolic subgroup generated by s_1.
The anchor coset representatives are: $x_{4k+1} = w^k$, $k = 0, \ldots, 5$.
And the spanning tree: $x_2 = \sigma_3$, $x_3 = \sigma_3\sigma_1$, $x_4 = \sigma_2$.
The (partial) coset graph is the following:



3 The choice of w_C

Let $\{b_i\}$ the basis of the Hecke algebra (we explain how we find this basis in the corresponding article and one can see it explicitly written in the GAP program, which corresponds to G_{12}).

The indices i of the b_i representing the conjugacy classes of G_{12} are

$$1, 25, 46, 36, 48, 23, 47, 44$$

in the order of the conjugacy classes in CHEVIE.

4 The canonical trace

Define $\tau\colon H(G_{12}) \to R$ as $\tau(\sum \alpha_i b_i) = \alpha_1$. Further calculations yield: $\det(\tau(b_ib_j)) = -b^{552}$.