

# The exceptional group $G_{22}$

## 1 Presentation

The group  $G_{22}$  admits the following presentation:

$$G_{22} = \langle s_1, s_2, s_3 \mid s_1^2 = s_2^2 = s_3^2 = 1, s_1 s_2 s_3 s_1 s_2 = s_2 s_3 s_1 s_2 s_3 = s_3 s_1 s_2 s_3 s_1 \rangle.$$

The Hecke algebra associated with  $G_{22}$  admits the following presentation:

$$H(G_{22}) = \langle \sigma_1, \sigma_2, \sigma_3 \mid \sigma_1 \sigma_2 \sigma_3 \sigma_1 \sigma_2 = \sigma_2 \sigma_3 \sigma_1 \sigma_2 \sigma_3 = \sigma_3 \sigma_1 \sigma_2 \sigma_3 \sigma_1, \sigma_i^2 = a\sigma_i + b, \quad i = 1, 2, 3 \rangle.$$

## 2 (Partial) action coset graph

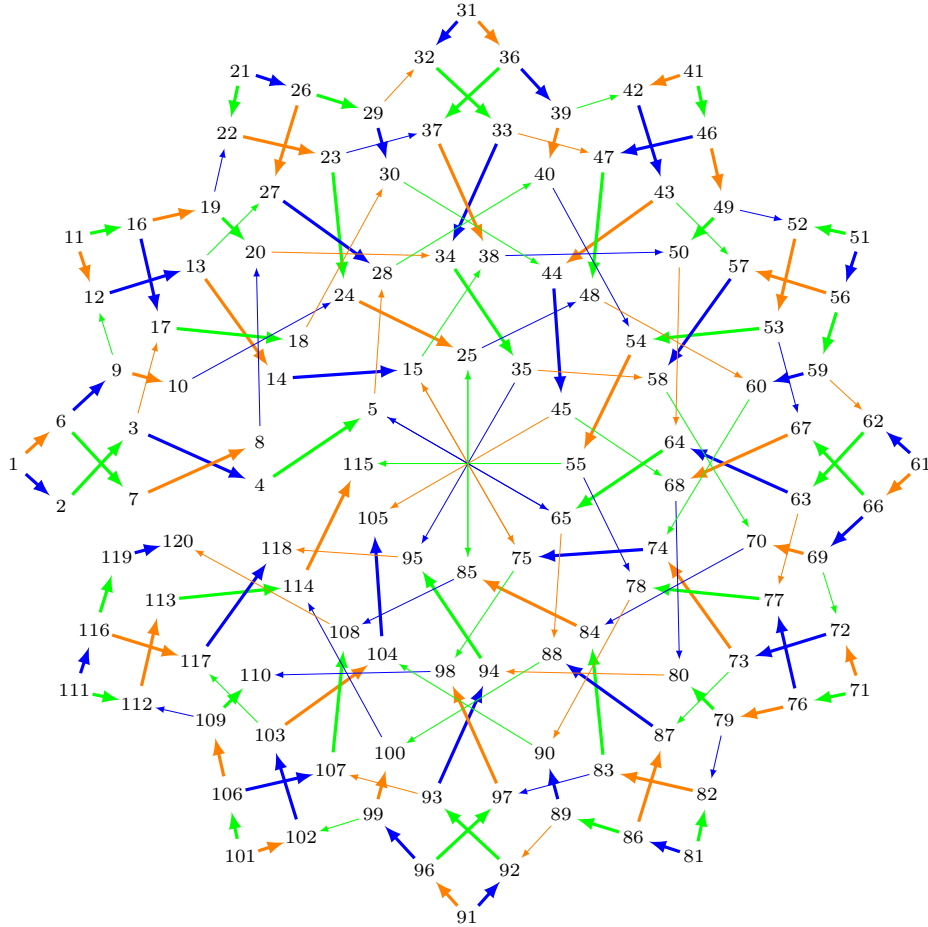
Let  $w = \sigma_1 \sigma_2 \sigma_3 \sigma_1 \sigma_2$ .

Let  $W'$  the parabolic subgroup generated by  $s_1$ .

The anchor coset representatives are:  $x_{10k+1} = w^k, k = 0, \dots, 11$ .

And the spanning tree:  $x_2 = \sigma_3, x_3 = \sigma_3 \sigma_1, x_4 = \sigma_3 \sigma_1 \sigma_3, x_5 = \sigma_3 \sigma_1 \sigma_3 \sigma_1, x_6 = \sigma_2, x_7 = \sigma_2 \sigma_1, x_8 = \sigma_2 \sigma_1 \sigma_2, x_9 = \sigma_2 \sigma_3, x_{10} = \sigma_2 \sigma_3 \sigma_2$ .

The (partial) coset graph is the following:



## 3 The choice of $w_C$

Let  $\{b_i\}$  the basis of the Hecke algebra (we explain how we find this basis in the corresponding article and one can see it explicitly written in the GAP program, which corresponds to  $G_{22}$ ).

The indices  $i$  of the  $b_i$  representing the conjugacy classes of  $G_{22}$  are

1, 61, 121, 181, 240, 230, 113, 173, 233, 239, 91, 151, 211, 238, 117, 177, 237, 216

in the order of the conjugacy classes in CHEVIE.

## 4 The canonical trace

Define  $\tau: H(G_{22}) \rightarrow R$  as  $\tau(\sum \alpha_i b_i) = \alpha_1$ . Further calculations yield:  $\det(\tau(b_i b_j)) = b^{7224}$ .